



VARIETIES OF REPRESENTABLE ALGEBRAS WITHOUT FINITE AXIOMATIZATIONS AND WITHOUT THE AMALGAMATION PROPERTY

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Received January 12, 2011; Re-revised November 11, 2017

Abstract

Let α be an ordinal and κ be a cardinal, both infinite, such that $\kappa \leq |\alpha|$. For $\tau \in {}^\alpha\alpha$, let $\text{sup}(\tau) = \{i \in \alpha : \tau(i) \neq i\}$. Let $G_\kappa = \{\tau \in {}^\alpha\alpha : |\text{sup}(\tau)| < \kappa\}$. We consider variants of polyadic equality algebras by taking cylindrifications on $\Gamma \subseteq \alpha$, $|\Gamma| < \kappa$ and substitutions restricted to G_κ . Such algebras are also enriched with generalized diagonal elements. Instances of such algebras are Halmos' quasipolyadic equality algebras and Lucas' extended cylindric algebras. We show that for any variety V containing the class of representable algebras and satisfying a finite schema of equations, V fails to have the amalgamation property. We then prove that the class of representable algebras, in the case of Halmos' quasipolyadic algebras and Lucas's extended cylindric algebras, cannot be axiomatized by a set of universal formulas containing only finitely many variables. We also show that any universal axiomatization of Halmos' quasipolyadic algebras must contain infinitely many diagonal elements. The latter result answers a question posed by Andr  ka.

Keywords and phrases: algebraic logic, polyadic algebras, amalgamation.

Pioneer Journal of
Algebra, Number
Theory and its
Applications

PSP Pioneer Scientific
Publisher